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ESTIMATION OF DELAYED NEUTRON EMISSION PROBABILITY AND DELAYED NEUTRON YIELD

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Abstract: Delayed neutron emission probabilities have been calculated for fission product nuclei with use of the improved gross theory, and fairly good results have been obtained. These delayed neutron emission probabilities have been combined with the fission yields in the recently-completed second version of the JNDC File to calculate the total delayed neutron yields. The total delayed neutron yields have also been calculated from experimental delayed neutron emission probabilities as well as from the delayed neutron emission probabilities obtained from the empirical formula by Kratz and Herrmann. These results for the total delayed neutron yields are compared with the recommended values by Tuttle.

(delayed neutron, delayed neutron yield, fission product, gross theory of β-decay, empirical formula)

I. Introduction

Delayed neutron emission is an interesting and important phenomenon. Its essential nature is now well understood, but its details for many nuclides are still not known enough. In this paper, we study the delayed neutron emission of fission product nuclides theoretically with the help of the gross theory of β-decay which has recently been improved.[1,2] We also study them with use of the empirical formula by Kratz and Herrmann.[3]

Here, we briefly outline the gross theory of β-decay. For more details see Refs. 1 and 2. The gross theory gives the strength function of nuclear β-decay $|M_\Omega(E)|^2$ as,

$$|M_\Omega(E)|^2 = \int_{\varepsilon_{min}}^{\varepsilon_{max}} D_\Omega(E,\varepsilon) W(E,\varepsilon) \frac{dn_1}{d\varepsilon} d\varepsilon \,. \tag{1}$$

Here, Ω denotes the type of β-transition, ε the single-particle energy, and E the negative of the transition energy. The function $D_\Omega(E,\varepsilon)$ is the single-particle strength function, $W(E,\varepsilon)$ is the weight function to take into account the Pauli principle, and $dn_1/d\varepsilon$ is the density of single-particle states.

Recently, the gross theory has been improved in three respects. The first improvement is to take into account the effect of UV factors of the BCS pairing theory, the second is a modification of the weight function $W(E,\varepsilon)$, and the third is modifications of the single-particle strength functions $D_\Omega(E,\varepsilon)$ for axial-vector type transitions. With use of this improved model, the newly calculated half-lives become shorter than the old ones in the region far from the β-stability line in better agreement with experiment.

In the next section, we calculate delayed neutron probabilities by the gross theory, and compare with the values recommended by Reeder.[4] In Section III we make calculations with the empirical formula,[3] in Section IV we calculate the total delayed neutron yield by various methods, and in the last section we give a conclusion.

II. Delayed Neutron Emission Probability Calculated by the Gross Theory

The delayed neutron emission can occur when some final states of β^--decay have excitation energies higher than the neutron separation energy S_n. Furthermore, if the excitation energies of some final states are higher than the two-neutron separation energy S_{2n}, delayed two-neutron emission can occur. In general, the probability of delayed emission of i neutrons is denoted by P_{in}, and, including the case i=0, we have

$$\sum_{i=0}^{\infty} P_{in} = 1 \,. \tag{2}$$

The average number of delayed neutrons per β-decay is expressed as

$$P_n = \sum_{i=1}^{\infty} i P_{in} \,. \tag{3}$$

In this section, we calculate the sum of P_{in} over i=1 to ∞ by the gross theory. It is given as

$$\sum_{i=1}^{\infty} P_{in} = \frac{\lambda_n}{\lambda_\beta}, \tag{4}$$

with

$$\lambda_n = \ln 2 \cdot \int_{S_n}^{Q} S_\beta(E_{exc}) \frac{\Gamma_n}{\Gamma_n + \Gamma_\gamma} f(Q - E_{exc}) \, dE_{exc}, \tag{5}$$

$$\lambda_\beta = \ln 2 \cdot \int_0^{Q} S_\beta(E_{exc}) f(Q - E_{exc}) \, dE_{exc}. \tag{6}$$

Here, E_{exc} is the excitation energy of the daughter nucleus, and λ_β is the decay constant. The allowed-equivalent total β strength function,[2] $S_\beta(E_{exc})$, can be obtained from the strength functions $|M_\Omega(E)|^2$ of the gross theory. As for the competition factor

$\Gamma_n/(\Gamma_n+\Gamma_\gamma)$, Takahashi[5] and Tachibana[6] evaluated it for some precursors. However, their method is hard to apply to all the precursors, and in this paper we simply put this factor equal to unity. Since the γ-ray emission mainly occurs in the energy region just above S_n, this approximation is relatively good when the decay has a large window $Q-S_n$.

The β-decay half-life $T_{1/2}$ is expressed as

$$T_{1/2} = \ln 2/\lambda_\beta \, . \tag{7}$$

Therefore, Eq.(4) is rewritten as

$$\sum_{i=1}^{\infty} P_{in} = \frac{T_{1/2} \cdot \lambda_n}{\ln 2}. \tag{8}$$

We use the strength function given by the gross theory to calculate λ_n, but, as for the half-life $T_{1/2}$, we have two choices. One is to use the value calculated from the gross theory, and the other is, if possible, to use the experimental value. We denote the result in the former choice as $\sum_i P_{in}^{(1)}$ and that in the latter choice as $\sum_i P_{in}^{(2)}$. In the calculations of $\sum_i P_{in}^{(1)}$ and $\sum_i P_{in}^{(2)}$, we need Q and S_n values as input data. For these quantities we use the mass table[7] and the mass formula[8] as used in Refs. 1 and 2.

Reeder recommended P_{1n} values for 77 precursors in the fission product nuclei by evaluating various experimental data.[4] We have calculated $\sum_i P_{in}^{(1)}$ and $\sum_i P_{in}^{(2)}$ values for the precursors whose half-lives have been measured and whose P_{1n} values are found in the recommendation by Reeder. From the calculation with the mass formula,[8] the energy windows for the delayed emission of two neutrons are known to be open for some of these precursors. However, these windows for two-neutron emission are so narrow that we can approximate as

$$\sum_{i=1}^{\infty} P_{in}^{(1)} \approx P_{1n}^{(1)}, \tag{9}$$

$$\sum_{i=1}^{\infty} P_{in}^{(2)} \approx P_{1n}^{(2)}. \tag{10}$$

In Figs.1 and 2, we show the ratios between calculated values and recommended ones for these quantities. We see from these figures that $P_{1n}^{(2)}$ are slightly better than $P_{1n}^{(1)}$ on the average. We also show in Fig.3 the ratios between the half-lives calculated by the gross theory and the experimental half-lives found in JNDC File II (Japanese Nuclear Data Committee FP Decay Data File, second version). It should be noted that the mark in Fig.3 at $Q \approx 13$ MeV is for the decay of a highly excited state of ^{131}In; this state is probably a state of seniority three, and then its decay should not be treated by the gross theory of Refs. 1 and 2.

III. Delayed Neutron Emission Probability by Empirical Method

There is another approach, an empirical method, to study the delayed neutron emission. In this section, we use the empirical function proposed by Kratz and Herrmann,[3]

$$P_n = a\left(\frac{Q-S_n}{Q-K}\right)^b , \tag{11}$$

$$K = \begin{cases} 0 & \text{even-even} \\ 13/\sqrt{A} & \text{odd-A} \\ 26/\sqrt{A} & \text{odd-odd} \end{cases} .$$

We use the same mass table and mass formula as used in Section II for obtaining the Q and S_n. We have determined the values of the parameters a and b from comparison with Reeder's P_{1n} values by the non-weighted least-squares method. The result is

$$a = 62.8 \, , \quad b = 3.37. \tag{12}$$

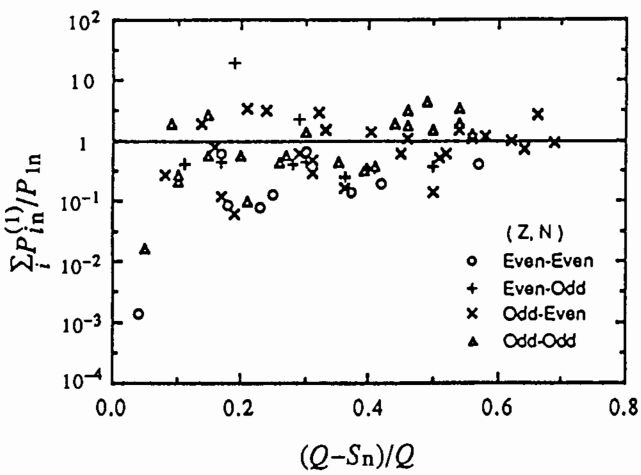

Fig.1 Ratios between the delayed neutron emission probabilities calculated by the gross theory (choice 1) and those recommended by Reeder.

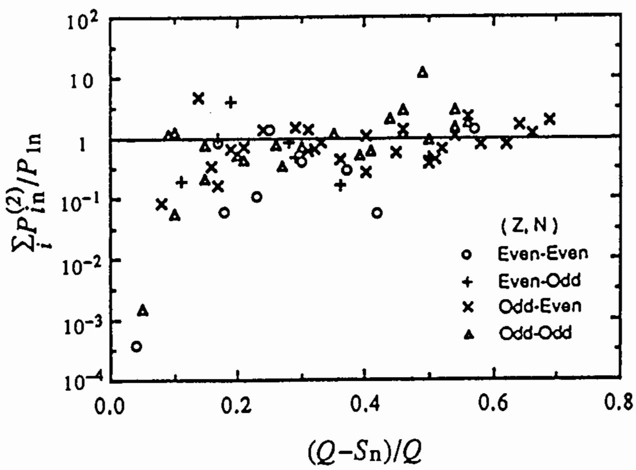

Fig.2 Ratios between the delayed neutron emission probabilities calculated by the gross theory (choice 2) and those recommended by Reeder.

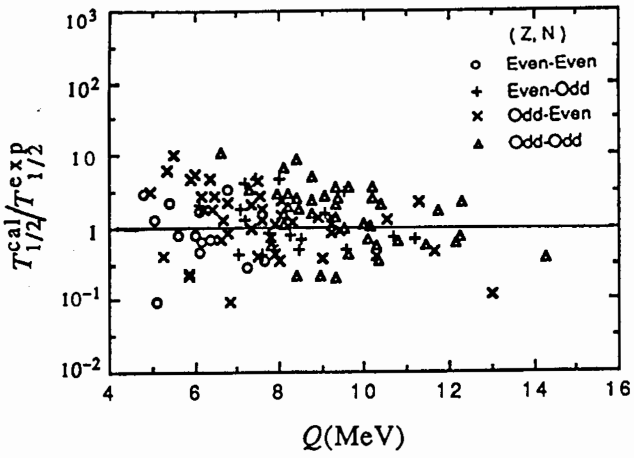

Fig.3 Ratios between calculated and experimental half-lives.

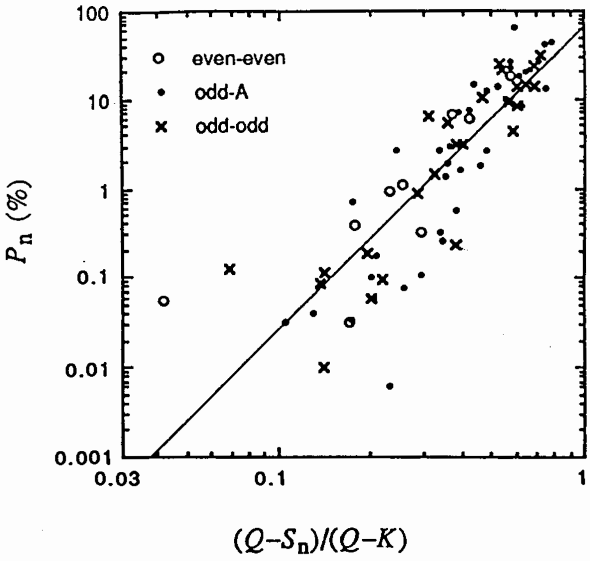

Fig.4 Plots of delayed neutron emission probabilities versus the quantity $(Q-S_n)/(Q-K)$. The line represents the empirical formula Eq.(11) with the parameter values given by Eq.(12).

The empirical formula Eq.(11) with the parameter values given by Eq.(12) is shown with a solid line in Fig.4. This formula is also used in the next section to calculate the total delayed neutron yield.

IV. Total Delayed Neutron Yield

In this section, integral calculations of the total delayed neutron yields ν_d are performed for 20 fissioning nuclei. It is given for each fissioning nucleus as,

$$\nu_d = \sum_{\mathrm{all FP}} Y_c P_n \, . \qquad (13)$$

The cumulative yields Y_c in the above equation are taken from JNDC File II. As for the P_n values, six data sets are used. Therefore, six values of ν_d are obtained for each fissioning nucleus. These six data sets of P_n are as follows:

1. Adopted values in JNDC File II
2. Recommended values by Reeder[4]
3. Evaluated values by Mann[9]
4. Evaluated values by Lund[9]
5. Calculated values from the improved gross theory
6. Calculated values from the empirical formula

We have estimated the effect of multi-neutron emission on ν_d, and have found that it is quite negligible because the yields Y_c are very small for the nuclei which undergo multi-neutron emission. Therefore, the data set 5 consists of the right-hand side of Eq.(8). As for $T_{1/2}$ to be used in Eq.(8) in this case, we use the experimental half-lives in JNDC File II, and, if they are not found in this file, the values calculated from the gross theory.

The calculated values of ν_d are given in Table I. In this table, we see that there are considerable differences among the values from different data sets in each fissioning nucleus, even among the data sets directly based on experimental data (Sets 2~4). Of the three experimental data sets, the set by Lund gives a considerably smaller value for every fissioning nucleus. It is also seen that the variations of the values from nuclide to nuclide are similar for all the data sets. Therefore, the discrepancy between the calculated and experimental ν_d values seems to be, at least partly, due to the cumulative yields. Especially, the yields in fast fission seem to be an issue of further research. There may also be problems in the measurement of ν_d, in particular, in the measurement of neutrons emitted very shortly after the fission.

V. Conclusion

We have calculated delayed neutron emission probabilities P_n of various nuclei by the gross theory and empirical formula. For nuclei having large energy windows, these calculations seem to give reasonable values of P_n. As for the total delayed neutron yields ν_d, there are fairly large discrepancies, indicating the necessity of further investigations.

Acknowledgments

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Talbe I. Comparison of calculated and recommended total delayed neutron yield ν_d. The values recommended by Tuttle[10] are given in the second column, of which the values in parentheses are those influenced by empirical estimations. The results calculated from the six data sets are given in the next six columns. Explanation of these six data sets is given in the text.

Nuclide	Tuttle's values	Calculated values					
		data set 1	data set 2	data set 3	data set 4	data set 5	data set 6
^{232}Th (F)	5.31 ±0.23	5.157	5.250	5.089	4.820	4.544	4.089
^{232}Th (H)	2.85 ±0.13	3.376	3.386	3.277	3.052	2.792	2.573
^{233}U (T)	0.667±0.029	0.884	0.905	0.890	0.812	0.736	0.758
^{233}U (F)	0.731±0.036	0.951	0.973	0.958	0.872	0.782	0.849
^{233}U (H)	0.422±0.025	0.739	0.754	0.741	0.661	0.566	0.658
^{235}U (T)	1.621±0.05	1.870	1.828	1.818	1.689	1.812	1.958
^{235}U (F)	1.673±0.036	2.057	2.068	2.039	1.880	1.860	1.956
^{235}U (H)	0.927±0.029	1.006	1.024	1.007	0.907	0.815	0.965
^{236}U (F)	(2.21 ±0.24)	2.365	2.354	2.329	2.143	2.236	2.356
^{238}U (F)	4.39 ±0.10	3.496	3.311	3.313	3.075	3.776	3.687
^{238}U (H)	2.73 ±0.08	2.738	2.633	2.623	2.426	2.568	2.851
^{237}Np (F)		1.310	1.293	1.285	1.144	1.239	1.401
^{239}Pu (T)	0.628±0.038	0.768	0.740	0.738	0.654	0.760	0.861
^{239}Pu (F)	0.63 ±0.016	0.726	0.695	0.693	0.601	0.678	0.819
^{239}Pu (H)	0.417±0.016	0.382	0.372	0.372	0.303	0.297	0.408
^{240}Pu (F)	(0.95 ±0.08)	0.909	0.874	0.873	0.772	0.904	1.060
^{241}Pu (T)	(1.52 ±0.11)	1.555	1.435	1.436	1.307	1.660	1.765
^{241}Pu (F)	(1.52 ±0.11)	1.484	1.359	1.363	1.234	1.520	1.682
^{242}Pu (F)	(2.21 ±0.26)	1.392	1.292	1.297	1.179	1.441	1.613
^{252}Cf (S)		0.664	0.575	0.587	0.526	0.676	0.739

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